Game Theory in Behavioral Science: How Bounded Rationality Transforms the Traditional Game-Theoretical Models

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Abstract

Economists and sociologists have applied classical and evolutionary game theory to the human context. Players' actions in laboratory experiments or daily life, however, do not always conform to predictions drawn from the theories, especially on an individual basis. Thus, in recent years, economists have focused greater attention on behavioral game theory, in which players are assumed to be boundedly rational. This paper aims to illustrate how bounded rationality is modeled in game-theoretic models, as well as how behaviors can be modeled in long-term games.

1 Introduction

Imagine going to a party where the following game was played. Each participant picked a number between 0 and 100. The person(s) whose number was the closest to seventy percent of the average would win a prize of a certain monetary amount. Which number should a player pick? Assuming the numbers selected were randomly distributed, the average would be around 50; thus a good choice would be 35. But other participants might recognize the same fact, so that the average would come out to be around 35, making 25 a good choice. Once again, however, other players would realize this too, so that now a player would want to pick 18, instead of 25. This process could continue until the outcome became 0, and consequently the iteration finally stopped. Suppose a player had learned game theory, assumed that players were fully rational, and chose the number 0. Surprisingly, the average came out to be 27. How would that happen? The reasoning process was impeccable in light of the Nash equilibrium principle, one crucial element in game theory, but it failed because people were not equally or fully rational. The above game scenario is one version of the classical game called beauty contest, which will be discussed more in a later section. When classical game theory is employed in the human behavioral context, we often encounter problems because of bounded rationality and other unforeseen factors. As a result, the study of behavioral game theory has emerged.

Game theory, a branch of applied mathematics, deals with strategies among rational players with respect to their preferences in game-theoretic models. The idea of games dates back to the 18th century, but the first formal publication was *Theory of Games and Economic Behavior*, by John von Neumann and Oskar Morgenstern in 1944 (Osborne, 2004, p. 2). Thanks to the rapid development of behavioral economics, newer work has shown that the classical game theory may not always indicate the strategy that a player ought to or will employ. In 1982, Maynard Smith published *Evolution and the Theory of Games*, which was a milestone in the development of evolutionary game theory (EGT). Smith observed in the preface that, "[p]aradoxically, it has turned out that game theory is more readily applied to biology than to the field of economic behaviour for which it was originally designed." Despite the fact that EGT was first designed for biological evolution, sociologists also have found the theory exceptionally valuable as they explore cultural and social evolution.

Camerer (1997) defined behavioral game theory as a theory that "aims to describe actual behavior, is driven by empirical observation (mostly experiments), and charts a middle course between over-rational equilibrium analyses and under-rational adaptive analyses" (p. 167). One of the main goals for behavioral game theorists is to "replace descriptively inaccurate modeling principles with more psychologically reasonable ones, expressed as parsimoniously and formally as possible" (Camerer, 1997, p. 185). And that goal is precisely the challenge.

Derived from classical game theory, behavioral game theory alters one major assumption of players from full rationality to bounded rationality. In addition, with a strong connection to evolutionary game theory, behavioral games can adapt the game models of biological evolution with appropriate modifications to help understand social or cultural evolution. The element that makes behavioral game theory unique is the human factor. Human beings do not always act in ways to maximize utility because humans are not capable of being fully rational. Therefore behavioral game theorists have set their focus on human behaviors in particular circumstances employing psychological and sociological theories. On the other hand and in the long run, human behavior does fall into certain patterns. Although researchers have, in some cases, used evolutionary game theory to analyze certain behavioral evolutions, long-term games are rarely presented explicitly in the studies so far. Thus I will attempt to discover appropriate evolutionary game-theoretic models for long-term human behavior framework.

2 Introduction to Game-theoretic Models

Game theory, as other sciences, contains a collection of models. The models consist of concise elements and reasonable assumptions to capture the essence of the situation and yet retain its simplicity. Because of the nature of behavioral game theory, we need to offer a brief overview of both classical game theory and evolutionary game theory.

2.1 Classical Game Model

Von Neumann and Morgenstern saw a decision under risk as a choice between lotteries with the decision maker trying to maximize the expected utility. To define rational decision maker, they suggested descriptive statements by means of six axioms. Earl Hunt (2007) models them after one presentation developed by R.D. Luce and H. Raiffa (pp. 140-141).

Axiom 1. All lotteries of the form (x, p, y) are in **A**.

Axiom 2. The relation $\succcurlyeq$ weakly orders all members of **X**.

Axiom 3. $((x, p, y), q, y) \approx (x, pq, y)$.

Axiom 4. $(x \approx y) \Rightarrow (x, p, z) \approx (y, p, z)$.

Axiom 5. $(x \succcurlyeq y) \Rightarrow (x \succcurlyeq (x, p, y) \succcurlyeq y)$.

Axiom 6. If $x \succcurlyeq y \succcurlyeq z$, then for some p, the relation $y \approx (x, p, z)$ holds.

The six axioms above assume that **X** is a set of possible outcomes of a decision, **A** is in the form of (x, p, y), x and y are both members of **X**, and p is a probability associated with x. For example, based on axiom 2, someone who has a weak preference ordering for transportation options so that $plane \succcurlyeq train\ and\ train \succcurlyeq automobile$, should also have the weak preference ordering, $plane \succcurlyeq automobile$. The implications are called comparability and transitivity. The symbolic expression for transitivity is:

$$(x \succcurlyeq y) \cdot (y \succcurlyeq z) \Rightarrow (x \succcurlyeq z).$$

Further, axiom 5 represents a situation similar to a possible preference for cakes. For example, someone who likes chocolate cake better than vanilla cake, will then like chocolate cake at least as much as any randomization between chocolate and vanilla cake, and like any randomization between chocolate and vanilla cake at least as much as vanilla cake.

In classical games, players are assumed to be fully rational in the sense of expected utility theory and to know the specification of the game. The expected utility is determined by the possible outcomes, or payoffs, and their corresponding probabilities. Suppose a lottery has a set of payoffs $x_1, x_2, ..., x_n \in \mathbb{R}$ with corresponding probabilities $p_1, p_2, ..., p_n \in [0,1]$ and $\sum_{i=1}^{n} p_i = 1$. The expected utility is defined to be $E = \sum_{i=1}^{n} x_i p_i$ (Gintis, 2000, p. 41). Notice that the expected utility is calculated based on payoffs, rather than monetary values, because utility functions for objective value are generally non-linear (Hunt, 2007, p. 143). Moreover, each player should have consistent preferences and mutually consistent beliefs. Although classical game theory has its limitations as it has been tested in laboratory experiments, it still offers a comprehensive framework. With "an appropriate set of Bayesian priors concerning the state of nature," boundedly rational individuals can be considered fully rational (Gintis, 2008, p. 2).

2.2 Evolutionary Game Model

An evolutionary game borrows from the idea of evolution by natural selection. The individual organisms or species, as players, possess sets of strategies (i.e. all of the feasible evolutionarily strategies) consisting of heritable phenotypes. Payoffs are measured in terms of fitness, which is defined as "the expected per capita growth rate for a given strategy and ecological circumstance" (Vincent & Brown, 2005, p. 16). The main distinction from classical game theory, according to Vincent and Brown, concentrates on the role of strategies. In evolutionary game theory, strategies are preserved over time through generations, while players inherit rather than choose their strategies. Occasionally mutations occur, producing novel strategies. Furthermore, in evolutionary game theory, players are in groups consisting of evolutionarily identical individuals; that is, individuals are grouped with others who have the same strategy set and the same expected payoffs. Another crucial distinction of evolutionary games is the role of nature. In classical games, rationality motivates players to choose their optimal strategies; whereas in evolutionary games, natural selection drives the biological evolution, functioning as the "agent of optimization."

The evolutionary game consists of an inner game and an outer game. The inner game involves only ecological processes while the outer game involves evolutionary processes (Vincent & Brown, 2005; T. L. and T. L. S. Vincent, 2000). The inner game corresponds to Darwin's second and third postulates—the struggle for existence and that heritable variation influences the struggle for existence. The outer game corresponds to Darwin's first postulate of heritable variation (Vincent & Brown, 2005, p. 85; T. L. and T. L. S. Vincent, 2000, p. 22). Consequently, the evolutionary game model contains two types of time scales—ecological time, T_{ec}, and evolutionary time, T_{ev}. The ecological time scale is associated with the time for the population density to return to ecological equilibrium—population dynamics—while the evolutionary time scale is associated with the time for the strategies to return to a strategy equilibrium—strategy dynamics (Vincent & Brown, 2005, p. 119).

2.3 Behavioral Game Model

As Camerer has observed, "game theorists fell in love with the elegance of their model; it spoke to them and they thought that's how people really behave.…Over time, we've started to move away from a hyper-rational model to a more psychologically nuanced model" (as cited in Robinson, 2004, p.3). Indeed, "no-regret" decisions rarely happen to humans. Here we adopt Gintis's notion (2008) of the beliefs, preferences, and constraints model that require only preference consistency (p. 2).

Again, payoffs are comparable. Therefore, we need to define a binary relation with a preference ordering $\succ_A$ on A with the following three properties (Gintis, 2008, p. 5):

1. Completeness: $x \succcurlyeq_A y$ or $y \succcurlyeq_A x;$
2. Transitivity: $x \succcurlyeq_A y$ and $y \succcurlyeq_A z$ $imply$ $x \succcurlyeq z;$
3. Independence from Irrelevant Alternatives: $For\ x,\ y \in B,$
   $x \succcurlyeq_B y \Leftrightarrow x \succcurlyeq_A y, \forall x, y, z \in A$ and $\forall B.$

The axiom of independence from irrelevant alternatives implies that the preference between two alternatives does not depend on choice sets. Although it is subtle and controversial, this behavioral assumption means that choice sets do not need to be specified.

Belief is incorporated into the model because it logically stands between decision and payoff. Classical game theory ignores the current status quo of players, but the payoffs for players depend on their current personal states and strategic circumstances. Thus, it is more plausible to introduce a decision maker's current status quo into the theory, although behavioral experiments have shown that bounded rational agents systematically violate the expected utility theory. That phenomenon does not imply that agents have inconsistent preferences over the appropriate choice dimension, but rather that they have incorrect beliefs given strategic circumstances (Gintis, 2008, p. 3).

2.4 Remarks

Most literature on behavioral game theory has focused on the incorporation of psychology and sociology rather than on the ecological aspect. Human interactions are, in fact, much like biological interaction. The ecological process often depends on the abundance of some resource (Vincent & Brown, 2005, p. 30). The environment gives feedback to organisms influencing population growth and favoring adaptive traits. Similarly, the social environment gives feedback to individuals influencing human behaviors and favoring traits that will optimize fitness in such a society. Social selection, therefore, plays a role like natural selection and drives social evolution. From the point of view of evolutionary game theory, a human behavioral game is made of an inner game and an outer game as well. The inner game refers to a game that is repeatedly played and reaches a particular equilibrium that forms the most adapted strategy at a certain period. The outer game refers to a game that is played over time and addresses the dynamics of the equilibrium in terms of the strategy frequencies. In social evolution, a novel strategy appears similar to a mutation in biological evolution. Equilibrium varies at different stages of history caused by limitations on sources and constantly changing strategic circumstances. The idea of an ecological and an evolutionary time scale works in the human behavioral context as well. Furthermore, a reasonable assumption about the application of these games to the human setting is that, in general, the evolutionary time scale is much slower than the ecological time scale; that is, most of the time, the dynamical social system will be near a slowing and changing ecological equilibrium solution. That assumption is consistent with the observation that "humans are much closer to time consistency, and have much longer time horizons than any other species" (Gintis, 2008, p. 12).

3 Rationality and Decision Making

Decision makers face bounded rationality when dealing with unique situations in the real world because of such limitations as response time, memory, available information, foresight, and reasoning. Many earlier, well-defined theories had flaws when applied to the development of behavioral science, but the discovery of the flaws represents great progress. Because human behavior is complex, various theories are necessary in distinct situations. We will look at decision making over time, risk and uncertainty first, and then further explore some other essentials that influence decision making.

3.1 Decision Making over Time

Individuals often have to deal with circumstances in which current decisions influence the choices available in the future. Also, an extended lag is typical between the time people make decisions and the time they experience the consequences of earlier decisions. For instance, people consider a 401(k) plan as a means to secure a stable retirement income. One has a small amount of money transferred from a current pay cycle to a monthly saving account. Although current utility may decline, future utility after retirement will potentially increase significantly. Alternatively, however, time inconsistency, which refers to people's inconsistent preferences over time, means that a person may sacrifice long-term payoffs for the sake of immediate payoffs. For example, a person who quits smoking will have better health, but because of the present urge (immediate payoff), keeps the smoking habit.

Several models are associated with time discounting. First is the exponential discounting model:

$$U(x) = x_0 + \delta x_1 + \delta^2 x_2 + \cdots + \delta^T x_T = \sum_{t=0}^{T} \delta^t x_t,$$

where δ is the constant discounting factor between 0 and 1, the discount function $D(t) = \delta^t$, and x_t is the instantaneous payoff at period t. Although the exponential discounting model offers a nicely convergent function, the assumption of a constant discounting factor limits its strength because laboratory experiments suggest that the discount rate has a tendency to decline in the future (Streich & Levy, 2007, pp. 201-205).

A second model is the hyperbolic discounting model, which is more appealing because of its steep discounting rate in the near future and declined discounting rate in the distant future. As Diamond and Vartiainen (2007) have argued, "people…tend to exhibit short-run impatience and long-run patience" (p. 87). A generalized hyperbolic discount function is:

$$D(t) = 1/\,(1 + \alpha t)^{\gamma/\alpha},$$

where α, $\gamma > 0$, α determines "how much the function departs from constant discounting" (Loewenstein & Prelec, 1992, p. 580) and γ is a time preference parameter positively related to the instantaneous discount rate (Laibson et al,. 1998, p. 100). The problem with a hyperbolic function is that many of them do not converge.

The third time discounting model is the quasi-hyperbolic discount model:

$$U(x)=x_0 + \beta\delta x_1 + \beta\delta^2 x_2 + \cdots + \beta\delta^T x_T = x_0 + \sum_{t=1}^{T} \beta\delta^t x_t.$$

This model is similar to the exponential discount model after the present period. Therefore, the quasi-hyperbolic discount function is convergent, but fails to reflect the declining nature of the discount rate. Researchers have found that it roughly approximates observed behavior in laboratory experiments, especially for the first few periods (Streich & Levy, 2007, p. 211).

Gintis (2008) used the experiment conducted by Ainslie and Haslam (1992) as an example. Subjects could accept one of two offers, either \$10 on the day of the experiment or \$11 a week later. In the second round, the same subjects could accept one of two offers, either \$10 a year later or \$11 a year and a week later. Many of the subjects decided to take \$10 right away and \$11 in a year and a week (Gintis, 2008, p. 9). Let x be to get \$10 at time t and y be to get \$11 at time $t + 7$, where t is in days. Thus we have $x > y$ when $t = 0$ and $x < y$ when $t = 365$. However, these two conditions cannot be true at the same time. Gintis suggests a way to make the time inconsistency disappear by moving the decisions up one more dimension. For example, let x_0 mean \$10 right away and x_{365} mean \$10 in a year from the experimental day, and similarly for y_7 and y_{372}. Then there is no conflict between $x_0 > y_7$ and $x_{365} < y_{372}$ (Gintis, 2008, p. 10).

In fact, when we take time-dependent preference into consideration, players are merely maximizing their utilities derived from consistent preference. Following one common hyperbolic discounting function,

$u(z_t) = {}^{z_t}/t + 1$, where z_t is the amount of money and t is time in days. Thus, $u(z_0) = 10/(0+1) = 10$ and $u(z_7) = 11/(7+1) = 1.375$, so $u(z_0) > u(z_7)$ and $u(z_{365}) = 10/(365+1) = 0.027$ and $u(z_{372}) = 11/(372+1) = 0.029$, so $u(z_{372}) > u(z_{365})$.

3.2 Decision Making over Risk and Uncertainty

Humans are rarely certain about things in the real life. For example, patients often cannot describe symptoms exactly; doctors and nurses cannot state observations perfectly, laboratory reports always contain some degree of error, physiologists lack a complete understanding of the human body, pharmacologists have no complete understanding of the mechanisms concerning the effectiveness of drugs, and no one can precisely determine one's prognosis (Szolovits, 1995, p. 111). Despite the uncertainties, doctors still have to recommend appropriate treatments, and patients still need to decide whether to follow doctors' recommendations.

Uncertainty typically involves exogenous and endogenous aspects. The former is resolved randomly by nature, and the latter is attributed to some other decision makers (Muthoo, 1999, p. 1). The general approach to explain how people make decisions under uncertainty with exogenous variables tends to rely on utility theory pioneered by von Neumann and Morgenstern, an approach employed in classical game theory. The theory and its axioms (see section 2.1) assume that the probabilities associated with each state are known to players. Players, in turn, do not have to form any beliefs concerning the distribution of the states of nature (Muthoo, 1999, p.4). Because of ignorance about beliefs, expected utility theory fails when individuals observe the states of nature differently from the given objective probabilities. Consider the Allais paradox as an example:

There is a two-choice situation in a game with prizes $x = \$2{,}500{,}000$, $y = \$500{,}000$, and $z = \$0$. The first is a choice between lotteries $\pi = y$ and $\pi' = 0.1x + 0.89y + 0.01z$.

The second is a choice between $\rho = 0.11y + 0.89z$ and $\rho' = 0.1x + 0.9z$. Most people, when faced with these two choice situations, choose $\pi \succ \pi'$ and $\rho' \succ \rho$ (Gintis, 2008, p. 17).

Based on the expected utility function, however, $u(\pi) = u(y), u(\pi)' = 0.1u(x) + 0.89u(y) + 0.01u(z), u(\rho) = 0.11u(y) + 0.89u(z)$, and $u(\rho') = 0.1u(x) + 0.9u(z)$. Consequently, $u(y) > 0.1u(x) + 0.89u(y) + 0.01u(z)$, which implies $0.11u(y) > 0.1u(x) + 0.01u(z)$, so $0.11u(y) + 0.89u(z) > 0.1u(x) + 0.9u(z)$, which means $u(\rho) > u(\rho')$. Players' beliefs towards utilities of lotteries are derived from a four-fold pattern of risk attitudes, namely "risk seeking for low-probability gains and high-probability losses, coupled with risk aversion for high-probability gains and low-probability losses" (Fox & See, 2003, p. 279). The fact that people tend to choose $\pi \succ \pi'$ and $\rho' \succ \rho$ most likely relates to risk aversion. One approach to cope with the inconsistent weighted probabilities is a weighted expected utility function developed by Chew and MacCrimmon:

$$\frac{\sum_{i=1}^{n} v(x_i)\, p_i}{\sum_{i=1}^{n} \tau(x_i)\, p_i}$$

where $v(x_i)$ is the evaluation function, $\tau(x_i)$ is a weighting function, and p_i is the probability of observing outcome x_i (Tuthill & Frechette, 2002; Machina, 2004). Using the Allais paradox as an example, the curves of indifference below are generated by three possible prizes, $z < y < x$, with corresponding probabilities, p_1, p_2, p_3, respectively. As $p_2 = 1 - p_1 - p_3$, all the prospects $(z, p_1; y, p_2; x, p_1)$ can be represented by the unit triangle in the (p_1, p_3)-plane. Under expected utility, indifference curves are linear and parallel; other the other hand, under weighted utility, indifference curves are linear as well, but not necessarily parallel, because curves become steeper as the probability of prize x rises and the probability of prize z falls (Quiggin, 1993, p. 39).

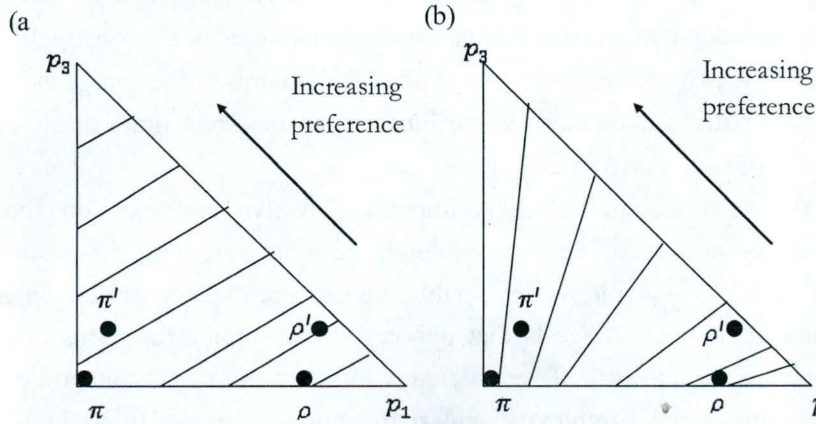

Figure 1: Indifference curves generated by expected utility (a) and weighted utility (b)

From the indifference curves, the expected utility predicts that $u(\pi') > u(\pi)$ and $u(\rho') > u(\rho)$, but weighted utility predicts that $u(\pi) > u(\pi')$ and $u(\rho') > u(\rho)$, which is consistent with the experimental observation.

3.3 Limitations of Reasoning Capacity

Given the complexity of human behavior, many factors affect decision making and include limited memory, cultural differences, different abilities, and limited response time. Memory also has a boundary or limit, both short-term and long-term; otherwise casinos would presumably face bankruptcy if gamblers were capable of counting down all the cards in blackjack. Memory limitations appear the most when players need to carry out successive actions as well as forecasting. In addition, memory involves personal beliefs that influence the way humans observe and convey information. Rubinstein (1998) claims that "[m]emory is a special type of knowledge….Imperfect recall is a particular case of imperfection of knowledge that lies at the heart of human processing of information" (p. 63). Memories are not only lost through the passage of time and the learning of new information, but they also get distorted. As memories are never exact reproductions of past events, forming memories is a reconstructive process, which makes memories vulnerable to misattribution; that is, "when a particular memory is attributed to an incorrect time, place, or person" (Okado & Stark, 2003).

Twelve researcher-led experiments in twelve countries on four continents and New Guinea explored cultural variations associated with such issues as ultimatums, public goods, and dictator games. Their combined research suggests that observable behavioral variations exist among the equilibria of different societies. Furthermore, market integration, anonymity, social complexity, and settlement size appear to be highly correlated across groups (Henrich, et al., 2005). Social behavior tends to be acquired through cultural learning and is much like evolutionary games in which the standards of fitness for organisms are shaped by nature. Consequently, individuals bring the preferences and beliefs that they have developed within particular social environments into the experimental situations.

With distinct knowledge and skills in the background, individuals are likely to have varied levels of information processing. Additionally, a specific response time will also limit an individual's information processing abilities. As a result, players may not want to investigate the overwhelming information in certain strategic circumstances. Instead, players would rather maintain their status quos, imitate successful agents whom they encounter, or adopt the recommended strategies of social norms.

3.4 Summary

The essentials above are by no means exhaustive; they merely help illustrate how and why decisions are influenced. Decision theory and relevant theories are largely employed in single (one-shot) games, which do not involve learning mechanisms. Unlike single games, repeated or evolutionary games do not emphasize individual strategies or decisions, as the equilibrium gets translated into population density or frequency of strategies, respectively. The beauty contest game represents a useful example (see introduction). Based on the experiment conducted by Ho, Camerer, and Weigelt, during the first round, the choices are "widely distributed and far from equilibrium" (Ho, et al., 1998), and the majority of the choices are around 21-40, which are about 1-3 levels of iterated rationality (Camerer, 2003, p. 18). But "choices converge toward the equilibrium point over time" (Ho, et al., 1998). As the players continue with the games, they observe the payoffs and adapt to the rules. And hence individual beliefs do not influence the strategies or decisions as strongly in long-run games; instead, "social norms" determine how reasonably rational players should behave.

4 Inner and Outer Games of Social Evolution

Individual interactions in repeated games are a learning process that constitutes the inner game of social evolution. The environment can be stationary or non-stationary in the inner game, whereas the environment should be dynamic in the outer game. In the inner game, equilibrium can be transformed into population dynamics, consisting of groups of individuals who adopt the same strategies, and population density. In the outer game, equilibrium can be transformed into strategy dynamics, or strategy frequency.

4.1 Inner Game

To study the inner game, economists have applied several learning mechanisms to the repeated games. In the reinforcement learning model, players associate payoffs with strategies according to experience and translate those payoffs into the probabilities of adopting those strategies in the future (Bell, 2001, p. 89). Economists find that reinforcement learning not only explains the existing equilibrium concept in game theory, but also provides an equilibrium selection technique in the presence of multiple equilibriums (Bell, 2001, p. 89). Models of learning explain the behaviors of players to some extent, but the models do not address population dynamics, the key feature of repeated games.

The primary tools in the study of population dynamics are the replicator equations that predict the distribution of certain genes or organisms at any given time in a particular environment (Gintis, 2000, p. 189). Under a structured pattern and featured environment, some replicators will have a better chance of survival, and those strategies will reproduce at an appropriately faster rate. The generally adopted replicator equation is

$$\dot{x} = [u(e^i, x) - u(x,x)]x_i, \text{ where } u(\dot{x}, x) = \sum_{i=1}^{k} x_i u(e_i, x).$$

The proportion of population that employs the pure strategy i at time t, is represented by the expression $x_i(t) = p_i(t)/p(t)$; the fitness of pure strategy i, is $u(e^i, x)$; and the population average payoff is $u(x, x)$ (Weibull, 1995, p. 72).

Human cultural evolution, however, has a less direct relationship to the pattern of replicators. Therefore, one assumption regarding cultural evolution is that individuals imitate the behaviors from other people; that is, culture changes by "transmission from others" (Gintis, 2000, p. 217). We can divide the people who are imitated into three groups: "parents (vertical transmission), peers (horizontal transmission), and influential individuals and institutions (oblique transmission)" (Gintis, 2000, p. 217). To modify the dynamic, evolutionary model for the social context, two new elements need to be added to the replicator equation, time rate and choice probability (Weibull, 1995, p. 152). Time rate, $r_i(x)$, is the average rate at which each individual reviews a strategy choice i. Choice probability,

$p_i^j(x)$, is the probability that each individual with strategy i will change to some other pure strategy j. Both of the elements are determined by the current performance of possible strategies and the current population state (Weibull, 1995, p. 152). Consequently, population dynamics for replication by imitation becomes:

$$\dot{x}_i = \sum_{j \in K} x_j r_j(x)\, p_j^i(x) - r_i(x) x_i$$

The inflow to the subpopulation i is $\sum_{j \neq i} x_j\, r_j(x)\, p_j^i(x)$, the outflow from subpopulation i is $\sum_{j \in K} x_i\, r_i(x)\, p_i^j(x)$, and the aggregate review times in the subpopulation i is $\sum_{j \in K} x_i\, r_i(x)\, p_i^j(x) = r_i(x)\, x_i$ (Weibull, 1995, pp. 152-153).

For example, in the case of considerate smoking behavior studied by Nyborg and Rege, the environment of the inner game could be stationary or non-stationary depending on whether the smoking policy, which is the driven power, is changing over time. If the environment is stationary, players will review their strategies regularly. If, on the other hand, the environment is non-stationary, individuals would want to review their strategies each time the policy is revised and include regular reevaluations of their strategies regardless of the policy modifications. Both the reviewing rate and probability of switching strategies are determined by the current environment and the current approval level of non-considerate smoking behavior.

4.2 Outer Game

The outer game of social evolution is much like biological evolution. According to Darwin, "one cannot explain the evolution of man and social faculties without a reference to natural selection and struggle for life"

(Marciano, 2007, p. 13). Both biological evolution and social or cultural evolution can be described as processes "of continuous adaptation to [unforeseeable] events, to contingent circumstances which could not have been forecast" (Hayek, 1988, p. 25; Marciano, 2007). Although social or cultural evolution can hardly be predicted, human behavior tends to fall into certain patterns. Many social conventions have two features of interest: "self-enforcing behavioral regularities" and conformity without thought to entrenched social structures (Epstein, 2001, p. 9). The general replicator equation or the modified version for replication, however, does not stress strategy dynamics, the fundamental characteristic of evolution. Therefore, we should adopt a new framework. Of the limited frameworks available, one is the G-function developed by Vincent and Brown. Although their approach is still controversial in the field of biological evolutionary games and is not widely adopted, the G-function provides relatively simple and generalized tools. The advantage of a G-function is that it could be used for a single population with multiple stages, or multiple populations with interactions (Vincent and Brown, 2005, p. 89).

A function $G(v, \boldsymbol{u}, \boldsymbol{x})$ is a fitness generating function (G-function) for population dynamics if and only if $G(v, \boldsymbol{u}, \boldsymbol{x})|_{v=u_i} = H_i(\boldsymbol{u}, \boldsymbol{x})$, $i = 1, \dots, n_s$, where $H_i(\boldsymbol{u},\boldsymbol{x})$ defines the fitness function determined by strategies $(\boldsymbol{u})$, population density $(\boldsymbol{x})$, and the number of possible strategies within the population n_s (Vincent and Brown, 2005, p. 77). Difference equations describe the population dynamics, while differential equations describe the continuous population dynamics (Vincent and Brown, 2005, pp. 34-38).

For populations with a single scalar strategy, population densities and strategies are represented by vectors $\boldsymbol{x} = [x_1 \cdots x_{ns}]$and $\boldsymbol{u} = [u_1 \cdots u_{ns}]$. Additionally, the expression for population dynamics for G-functions is represented by $x_i(t + 1) = x_i[1 + G(v, \boldsymbol{u}, \boldsymbol{x})|_{v=u_i}]$ for discrete games;

and by $x_i = x_i G(v, \boldsymbol{u}, \boldsymbol{x})|_{v=u_i}$ for continuous games, where $i = 1, ..., n_s$. In this case, the behavior of each population group must be dependent on the same "evolving trait" (Vincent and Brown, 2005, pp. 92-93). For example, if the number of enrollments in graduate schools entirely depends on the availability of jobs in the current market, then that parameter can be considered as the evolving trait in the behavioral model for graduate school enrollments.

For populations with vector strategies, population densities and the vector strategy of a population x_i are represented by vectors $\boldsymbol{x} = [x_1 \cdots x_{ns}]$ and $\boldsymbol{u}_i = [u_{i1} \cdots u_{inu}]$, where n_u is the number of "traits" in the vector $\boldsymbol{u}_i$. Moreover, population dynamics for G-functions are $x_i t(t + 1) = x_i[1 + G(\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{x})|_{v=u_i}]$ for discrete games and $\dot{x}_i = x_i G(\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{x})|_{v=u_i}$ for continuous games, where $i = 1, ..., n_s$. In this case, behavior of each population group can be determined by multiple "evolving traits" (Vincent and Brown, 2005, pp. 93-95). For example, if the number of enrollments in graduate schools depends on the job availability in the current market, the number of enrollments in undergraduate schools, and the number of programs in average graduate schools, then these three parameters could make up the vector strategies in the behavioral model for graduate school enrollments.

In a behavioral model, if the dynamic of resources is explicitly indicated, then the resource vector $\boldsymbol{y} = [y_1 \cdots y_{ny}]$, where n_y is the number of resources, needs to be included in the G-functions. Thus, the expression for population dynamics becomes $x_i(t + 1) = x_i[1 + G(\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{x}, \boldsymbol{y})|_{v=\boldsymbol{u}_i}]$ for discrete games and $\dot{x}_i = x_i G(\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{x}, \boldsymbol{y})|_{v=\boldsymbol{u}_i}$ for continuous games, where $i = 1, ..., n_s$. The dynamics equations for resource vector $\boldsymbol{y}$, in dimension n_y, are expressed by resource function $\boldsymbol{N}(\boldsymbol{u}, \boldsymbol{x}, \boldsymbol{y})$ and defined as $\boldsymbol{y}(t + 1) = \boldsymbol{y} + \boldsymbol{N}(\boldsymbol{u}, \boldsymbol{x}, \boldsymbol{y})$ for discrete games and $\dot{y} = \boldsymbol{N}(\boldsymbol{u}, \boldsymbol{x}, \boldsymbol{y})$ for continuous games (Vincent and Brown, 2005, pp. 96-97).

The complexity of the social context can create a highly complicated G-function. For instance, if not all individuals are evolutionarily identical, we will need to introduce multiple G-functions. Besides the G-function approach, other methods, such as adaptive dynamics, are available for evolutionary games. Unfortunately, none of the methods has been universally adopted. The study of a theoretic-game model for evolutionary games is still in progress, in particular, the idea of applying game models to behavioral evolution. The field, therefore, offers many prospects for future study.

5 Conclusion

Behavioral game theory is still in its preliminary developmental stage; the framework is not yet rigorously established. The psychological aspect of behavioral games has been studied by means of the combination of game-theoretic models and empirical results. A variety of mathematical applications are employed under certain circumstances in one-shot games, but the "real situations" are hard to capture because there are still unknown elements that cause the results to diverge from empirical outcomes. Also, researchers have applied replicator equations to social contexts without formally categorizing the games as evolutionary inner or outer games. The connection between social evolution and biological evolution is widely observed. Economists and biologists have been improving the framework of evolutionary game theory for biological evolution, and thus it may enhance the study of social evolution. To further develop behavioral game theory, game theorists will need to work closely with neuroeconomists, sociologists and psychologists.

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